International Journal of Control, Energy and Electrical Engineering (CEEE)
Vol.6 pp.10-16

Singularities Free for Attitude Tracking via Quaternion based Robust Control

Mouad Kahouadji, Amal Choukchou-Braham, M Rida Mokhtari and Brahim Cherki
Laboratoire d'Automatique de Tlemcen LAT
Université de Tlemcen, Algeria
Email: mouad.auto@gmail.com

***Abstract*—This paper presents a nonlinear robust control based on Super Twisting algorithm for quadrotor's attitude tracking and stabilization. The model used in the control design is obtained using the quaternion representation with the aim to avoid singularities. The finite time convergence and stability of the closed loop system is proved through Lyapunov function candidate. The experimental results show the robustness and finite time convergence of the control in presence of parameters uncertainties and external disturbances.**

Index - Super Twisting algorithm, quaternion representation, Lyapunov stability, finite time convergence, quadrotor plateform.

I. Introduction

Unmanned aerial vehicles (UAVs) are a part of the future. They are being used more often for military and civilian purposes such as traffic monitoring, patrolling for forest fires, surveillance, and rescue, in which risks to pilots are often high.
The most used class of the UAV is the quadrotor, it has an evident advantage comparing to the other classes for various applications because of its vertical landing/take-off capability, payload, great maneuverability and easy to manufacture.
For this, the quadrotor becomes an interesting area of research. Various methods are developed to control the quadrotor position. We find the linear algorithms which deal with the system locally around its equilibrium point as LQR [1] and PID [2].
Since the quadrotor is an underactuated system, that is it has six degrees of freedom to be controlled and only four inputs, and highly coupled model, nonlinear controls were taken into consideration to deal with this latter. Serval approaches are developed in this direction such as backstepping [3], sliding mode control [4], Super twisting control [5] and adaptive backstepping [6]. The used representation in most researches is based on Euler angle which leads to control loss when the quadrotor passes by the position ($\phi = \pi/2$ or $\theta = \pi/2$ or $\psi = \pi$). We call this problem singularities. Quaternion based representation is the solution to avoid this problem, there are researches that have take attention to this representation where they developed various control such as chen and Lo [7] , Taybi [8] and Wu Shunan [9].

This paper presents the second order sliding mode control Super Twisting algorithm (STWA) for quaternion-based spacecraft attitude tracking and stabilisation. In our work we define firstly in section II, the quadrotor model using the quaternion representation associated with the fixed and mobile frames, where we get the kinematic and dynamic equations.
in section III, we compute the control law by defining a sliding variable and using the STWA.
Finally, we present experimental results that show the validation of the theoretical conclusions about stability, robustness and finite time convergence.

II. Mathematical Model of Quadrotor

The quadrotor consists of a rigid cross airframe with four individual rotors as seen in Fig1. The front and rear rotors, numbered 1 and 3, rotate counterclockwise (positive about the z-axis), while the left and right rotors, numbered 2 and 4, rotate in a clockwise direction. Vertical motion is achieved by increasing or decreasing the speed of each rotor by the same proportion. The roll motion is controlled by increasing the thrust of rotor 2 (4) and decreasing the thrust of rotor 4 (2) to obtain a positive (negative) roll to the right (left). The pitch motion is achieved similarly by differential speed between rotors 1 and 3. The yaw motion of the quadrotor is achieved by adjusting the average thrust of the clockwise and counterclockwise rotating rotors. When a yaw motion in the positive direction is desired for example, the rotor pair 1 and 3 increase by the same proportion, while the rotor pair 2 and 4 decrease by the same proportion. This will maintain the same overall aircraft thrust without pitching or rolling the aircraft.

To get the attitude equations of the UAV there are different ways. In our work we are intersting by the quaternion representation to avoid singularities. The quaternion is given by:

$$Q = q_0 + q_1 i + q_2 j + q_3 k \tag{1}$$

where q_0 is a scalar and $q_1 i + q_2 j + q_3 k$ is a vector.
The quaternion is described by the following propriety :

$$\|Q\| = q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \tag{2}$$

Let us consider the quadrotor as being a rigid body under external forces applied to its center of mass, the dynamic equation referred to the body coordinates system under the

Copyright IPCO-2018
ISSN 2356-5608

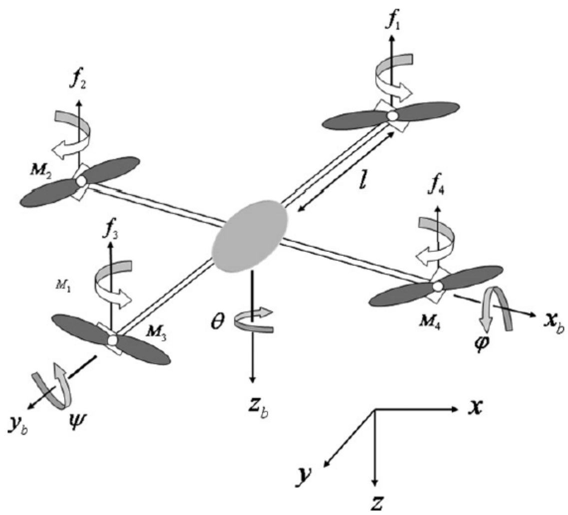

Fig. 1. Quadrotor model

Newton-Euler formulation is :

$$\begin{bmatrix} mI_{3\times3} & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} \omega \times mV \\ \omega \times J\omega \end{bmatrix} = \begin{bmatrix} u_z \\ u \end{bmatrix} \quad (3)$$

Using the kinematic and dynamic equations, quaternion representation are given by :

$$\dot{P} = q \otimes V \otimes \bar{q} \quad (4)$$

$$m\dot{V} = -mgz_b + q \otimes \begin{bmatrix} 0 \\ 0 \\ u_z \end{bmatrix} \otimes \bar{q} - [\omega\times]mV \quad (5)$$

$$\dot{Q} = \frac{1}{2}S(Q)\omega \quad (6)$$

$$\dot{\omega} = J^{-1}(-[\omega\times]J\omega + u + d) \quad (7)$$

where d is the bounded disturbance.

$$S(Q) = \begin{bmatrix} q_0 I_{3\times3} + [q\times] \\ -q^T \end{bmatrix} \quad (8)$$

$[q\times]$ is a skew matrix defined by :

$$[q\times] = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \quad (9)$$

$V = [v_x \quad v_y \quad v_z]$ is the translation velocity.
$\omega = [\omega_x \quad \omega_y \quad \omega_z]$ is the angular velocity vector;
$\bar{u} = [u \quad u_z] = [u_1 \quad u_2 \quad u_3 \quad u_z]$ is the control vector, that are the torques and the thrust forces generated by the four DC motors and is given by :

$$u_1 = \tau_\phi = lb(\Omega_2^2 - \Omega_4^2) \quad (10)$$

$$u_2 = \tau_\theta = lb(\Omega_1^2 - \Omega_3^2) \quad (11)$$

$$u_3 = \tau_\psi = \bar{\rho}(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \quad (12)$$

$$u_z = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2)$$

Ω_i are the angular speed of the four rotors respectively.

Let us introduce now the motor dynamique which contains electrical and mechanical equations. This model is composed of the series of a resistor $R[\Omega]$, an inductor $L[H]$ and a voltage generator $e[V]$. The resistor represents the Joule loss due to the current flow into the copper conductor. Its value depends on geometric and material characteristics such as wire resistivity, length and section.

The equations describing the motor are given by :

$$v = \frac{di}{dt} + Ri + K_e\Omega \quad (13)$$

$$J_m\dot{\Omega} = C_{em} - C_r \quad (14)$$

The inductor part is neglected because it is small and the electric part is so faster then the mechanical one; so the model will be as :

$$v = Ri + K_e\Omega \quad (15)$$

$$J_m\dot{\Omega} = -\frac{K_M K_e}{R}\Omega + \frac{K_M}{R}v \quad (16)$$

v is the voltage input: the real input of the system, K_M, K_e are mechanic motor constant and electric motor constant respectively and R is the motor resistance.

III. Control Design

To design the control law, we use the second order sliding mode algorithm called Super Twisting. The control goal is to get a good performance in term of stabilisation and attitude tracking.
Let consider the following sliding variable :

$$s = e_\omega + \lambda e_q \quad (17)$$

Where $\lambda = [\lambda_{q1} \quad \lambda_{q2} \quad \lambda_{q3}]$ is a constant gain vector.
e_q the quaternion error given by :

$$e_q = q - q_d \quad (18)$$

q_d is the desired position in the quaternion frame.
e_ω is the angular velocity error.

$$e_\omega = \omega - \omega_d \quad (19)$$

ω_d is the desired angular velocity.

To complete the control design we use two properties related to the quadrotor motion equation that is given as follow [7] :
Property 1 :

Copyright IPCO-2018
ISSN 2356-5608

International Journal of Control, Energy and Electrical Engineering (CEEE)
Vol.6 pp.10-16

The matrix $S(Q)$ has the following properties :

$$\begin{aligned} S(Q)^T S(Q) &= I_{3\times3} \\ \|S(Q)\| &= 1 \\ \frac{d}{dt}[S(Q)^T \dot{Q}] &= S(Q)^T \ddot{Q} \\ \|\omega\| &= 2\,\|S(Q)\| \end{aligned}$$

Using (3) and the previous properties, the desired angular velocity can be expressed as follow :

$$\omega_d = 2S(Q)^T \dot{Q}_d \tag{20}$$

$$\dot{\omega}_d = 2S(Q)^T \ddot{Q}_d \tag{21}$$

So the dynamic of sliding variable can be written as :

$$\dot{s} = \dot{\omega} - \dot{\omega}_d + \lambda(\dot{q} - \dot{q}_d) \tag{22}$$

The super twisting controller is given as follow :

$$u = J(J^{-1}[\omega\times]J\omega - J^{-1}d + \dot{\omega}_d - \frac{1}{2}\lambda S(Q)\omega + \lambda\dot{q}_d + z) \tag{23}$$

Where z is the super twisting correcting term expressed by:

$$z = -k_1\,|s|^{\frac{1}{2}}\,sign(s) - k_2 \int_0^t sign(s(\tau))d\tau + \nu \tag{24}$$

Where $k_1 = [k_{11} \quad k_{12} \quad k_{13}]$ and $k_2 = [k_{21} \quad k_{22} \quad k_{23}]$ are positive gains.
The closed loop sliding variable by introducing the control (23) is rewritten as :

$$\dot{s} = -k_1\,|s|^{\frac{1}{2}}\,sign(s) + \mu \tag{25}$$

$$\dot{\mu} = -k_2 sign(s) + \varrho \tag{26}$$

$\varrho = \dot{\nu}$ represents the dynamic of parameter uncertainties and external disturbances.

IV. Stability Analysis

To analyse the stability of the closed loop sliding variable, we propose the following state vector :

$$[x_1 \quad x_2] = [s \quad \mu] \tag{27}$$

Considering this variable changement, (25,26) can be expressed as :

$$\begin{aligned} \dot{x}_1 &= -k_1|x_1|^{1/2}sign(x_1) + x_2 \\ \dot{x}_2 &= -k_2 sign(x_1) + \varrho \end{aligned} \tag{28}$$

We used the follwing Lyapunov function candidate to demonstrate the stability of the closed loop system [15]:

$$V(x) = 2k_2|x_1| + \frac{1}{2}x_2^2 + \frac{1}{2}(k_1|x_1|^{1/2}sign(x_1) - x_2)^2 \tag{29}$$

This function is continuous and diffrentiable except at $x_1 = 0$.
The equation (29) can be rewritten on quadratic form as :

$$V(x) = \zeta^T P \zeta \tag{30}$$

Where

$$\zeta^T = [|x_1|^{1/2} sign(x_1) \quad x_2] \tag{31}$$

$$P = \frac{1}{2}\begin{bmatrix} 4k_2 + k_1^2 & -k_1 \\ -k_1 & 2 \end{bmatrix} \tag{32}$$

Using the theorem defined in Morino's paper [15] we find that :

$$\lambda_{min}(P)||\zeta||_2^2 \leq V(x) \leq \lambda_{max}(P)||\zeta||_2^2 \tag{33}$$

Where $||\zeta||_2^2 = |x_1| + x_2^2$ is the Euclidean norm of ζ.
λ_{min} λ_{max} are respectively the minimum and maximum eigenvalues of the matrix p.
The derivative of Lyapunov candidate function is :

$$\dot{V}(x) = -|x_1|^{-1/2}\zeta^T Q\zeta + \varrho R^T \zeta \tag{34}$$

Where $R^T = [-k_1 \quad 2]$
We propose the bounds of perturbation defined by the following :

$$\varrho(t,x) \leq h_{max} \tag{35}$$

Where h_{max} is a positive gain.
Using (35) we find :

$$\dot{V}(x) \leq -|x_1|^{-1/2}\zeta^T \bar{Q}\zeta \tag{36}$$

Where

$$\bar{Q} = \frac{k_1}{2}\begin{bmatrix} 2k_2 + k_1^2 - 2h_{max} & -k_1 \\ -(k_1 + \frac{2h_{max}}{k_1}) & 1 \end{bmatrix} \tag{37}$$

$\dot{V}$ is negative definite if $\bar{Q} > 0$. that is true if :

$$k_1 > 0 \tag{38}$$

$$k_2 > \frac{h_{max}\|R\|}{\frac{1}{2}k_1^2} \tag{39}$$

We have

$$|x_1|^{1/2} \leq \|\zeta\|_2 \leq \frac{V^{1/2}(x)}{\lambda_{min}^{1/2}\{P\}} \tag{40}$$

From (33), (36) and (40) we find that :

$$\dot{V} \leq -\gamma V^{1/2}(x) \tag{41}$$

Where

$$\gamma = \frac{\lambda_{min}^{1/2}\{P\}\lambda_{min}\{\bar{Q}\}}{\lambda_{max}\{P\}} \tag{42}$$

The previous result guarantees convergence of the vector $[s \quad \mu]$ to zero in finite time that is given by :

$$T_r = \frac{2V^{1/2}(x_0)}{\gamma} \tag{43}$$

Copyright IPCO-2018
ISSN 2356-5608

V. Experemental results

In this work we use a Qanser platform to validate our control. It contains a fixed quadrotor connected with a control card (Q8 usb) and powered by an amplifier as shown in Fig.2,3. The used sensors are encoders to mesure the angle positions and the actuators are DC motors (Motor-Pittman 9234S004). Experimental testing has been performed using Q8 usb card in combination with Matlab-Simulink- that allows us a real time visualisation and interaction. The four DC motors are powered by Quanser linear voltage amplifier driven by PWM signals.

Fig. 2. Quanser quadrotor

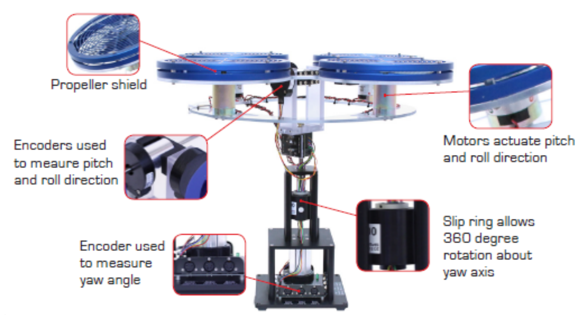

Fig. 3. Quanser quadrotor

We are carrying out the control signal via Simulink in block diagram format using the physical parameters given in table I and we obtain the following results due to many tests to show the performances of the proposed control.
Since, we are dealing with the attitude stabilisation and tracking of a fixed quadrotor platform, we suppose that the thrust force u_z is constante to compensate gravity force.
The gain values used in the next expirement are described in the table II :

A. Attitude Stabilisation

Figure (4) represents the quaternion response. This test has been realised by giving the quadrotor initial conditions that are equivalent to a Euler angles values as it is shown in figure (5). The initial conditions are ($q_0 \approx 0.96$, $q_1 \approx 0.2$, $q_2 \approx 0.2$,$q_3 \approx -0.1$) corresponding to ($\theta = 18°$; $\phi = 18°$; $\psi = -9°$). The obtainted results show the power of the control in term of stabilisation.

TABLE I
Quanser parameters

parameter	description	value	Unit
m	mass	2.85	Kg
l	distance between Pivot to each Motor	0.1969	m
b	thrust factor	2.98e-6	N/V
$\bar{\rho}$	drag factor	1.14e-7	
J_x	Roll inertia ϕ	0.0552	kgm^2
J_y	Pitch inertia θ	0.0552	kgm^2
J_z	Yaw inertia ψ	0.1104	kgm^2

TABLE II
Control gains

parameter	value	parameter	value	parameter	value
λ_{q_1}	2	k_{11}	200	k_{21}	40
λ_{q_2}	2	k_{12}	200	k_{22}	50
λ_{q_1}	1.5	k_{13}	150	k_{23}	20

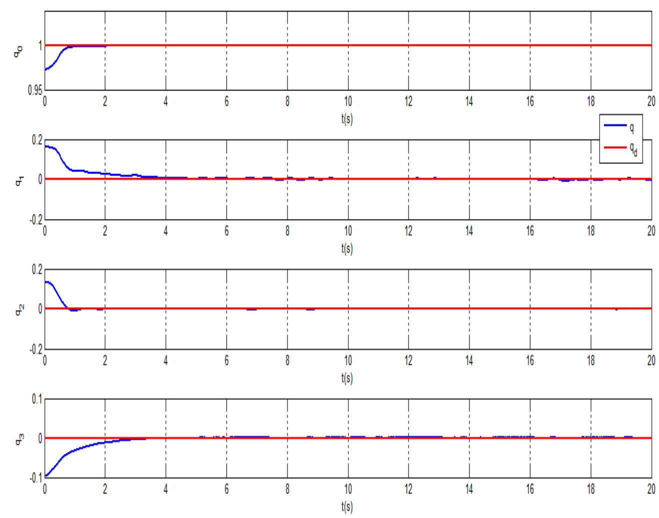

Fig. 4. Quaternion trajectories for stabilisation test

B. Attitude Tracking

In this experiment, we give the machine a desired trajectories with sinusoidal form. Figures (6,7) show the high performance of the control in term of tracking. The small oscillations in the response trajectories are due to the sensor noise and drift.

C. Robustness Test

Figures (8,9) present the quaternion and the corresponding Euler angles tracking responses. Where we applied a manual external force considered as external perturbations.

Copyright IPCO-2018
ISSN 2356-5608

International Journal of Control, Energy and Electrical Engineering (CEEE)
Vol.6 pp.10-16

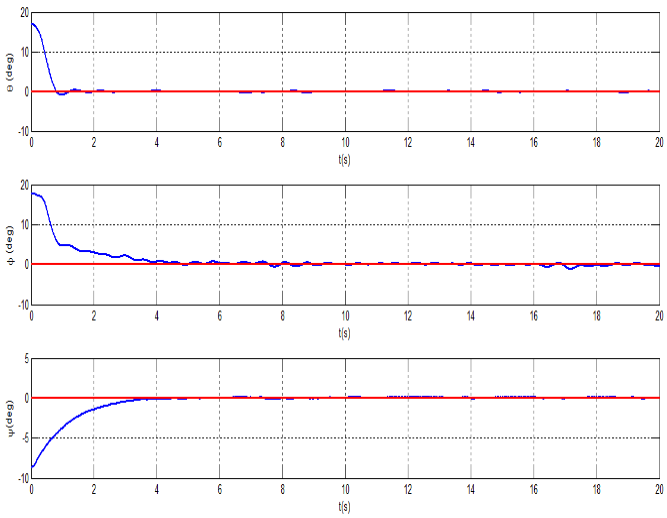

Fig. 5. Corresponding Euler angles trajectories for stabilisation test

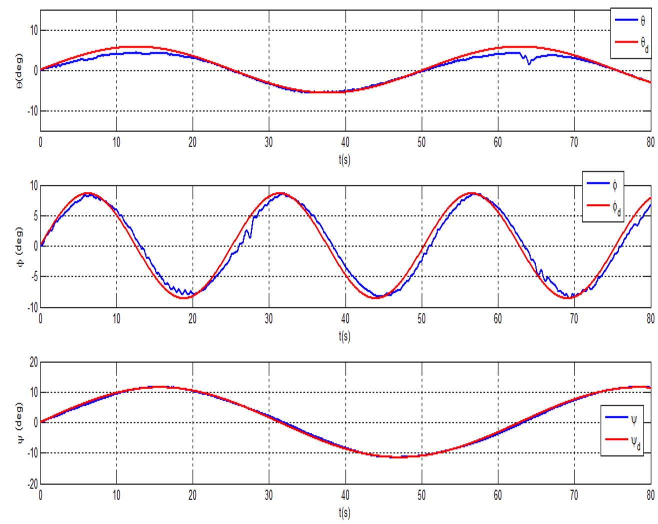

Fig. 7. Corresponding Euler angles trajectories for tracking test

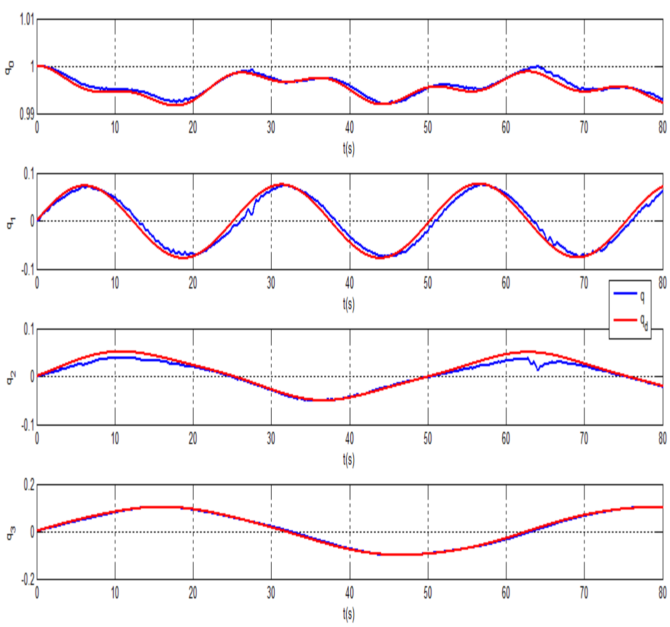

Fig. 6. Quaternion trajectories for tracking test

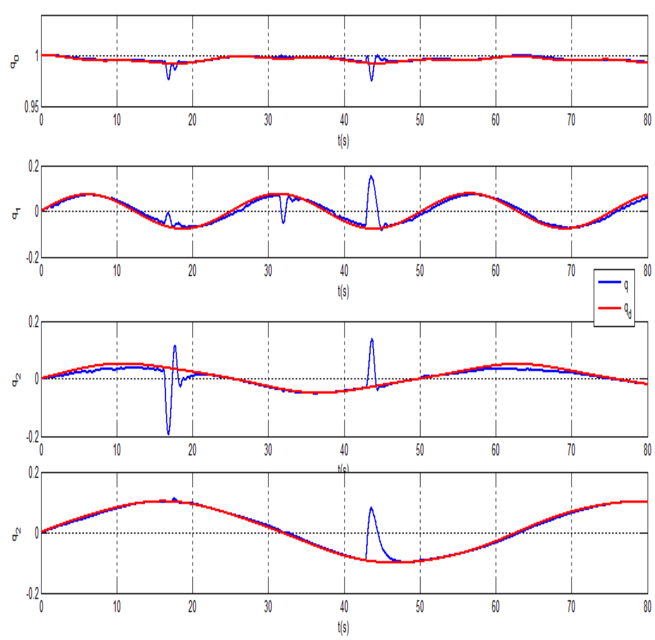

Fig. 8. Quaternion trajectories for robustness test

The disturbances was applied at $t = 15s$ on the θ orientation, at $t = 30s$ on the ϕ axes and at $t = 45s$ in both orientations. The effectiffness of the super twisting appears in this experience. In fact, we clearly see that there is a good rejection of the external diturbances and a quick reaction, which means that the control signal intervene so that the output follows the desired trajectory.

D. Singularities Avoidance Test

To show the powerful of quaternion representation with respect to Euler angles repesenation, we give to the quadrotor initial conditions that cause singularities. That is, $\theta, \Phi = \frac{\pi}{2}$ and $\psi = \pi$.

However, the Quanser experimental plateform is a fixed quadrotor, the physical limits due to this material don't allow to reach these angles except for the yaw angle.

This is why, we give the simulation results assiociated with the control based on Euler angles representation Fig.10 and with the quaternion representation Fig.11.

Experimental results only concerns the yaw angle and are given in Fig.12,13.

From these figures, it appears that the control based on quater-

Copyright IPCO-2018
ISSN 2356-5608

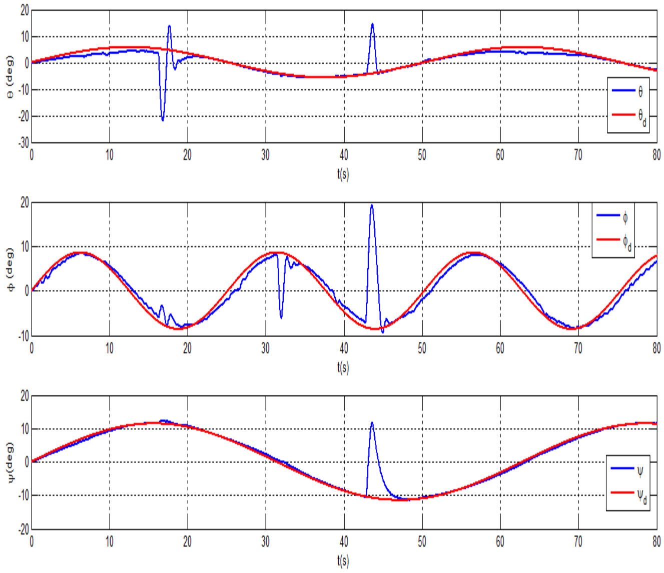

Fig. 9. Corresponding Euler angles trajectories for robustness test

nion represenattion can avoid sigularities, this overcomes the principal inconvenience for quadrotor control.

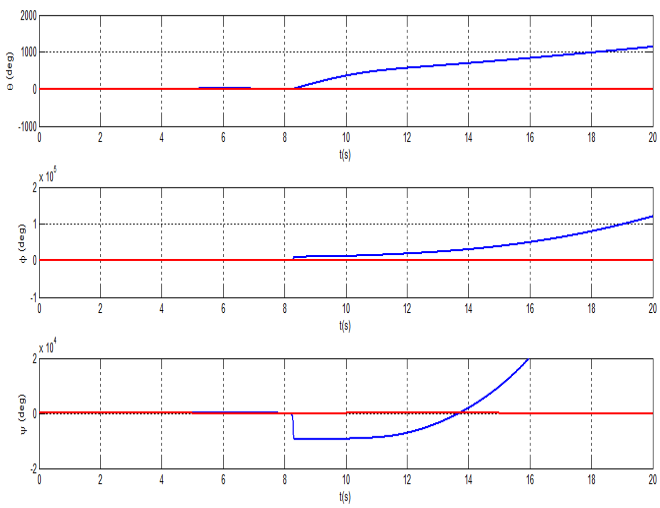

Fig. 10. Euler angle trajectories with control based on Euler representation

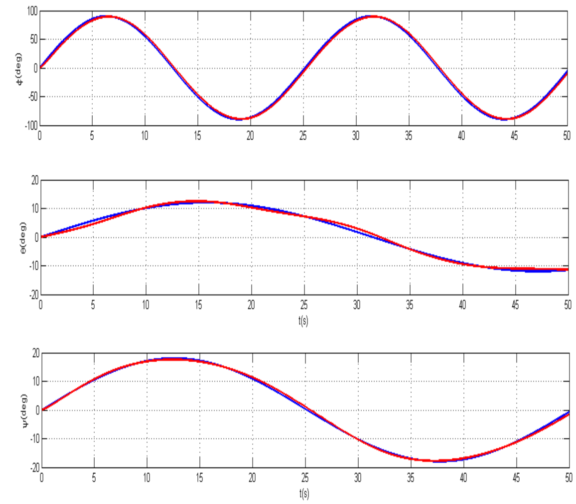

Fig. 11. Euler angle trajectories with control based on quaternion representation

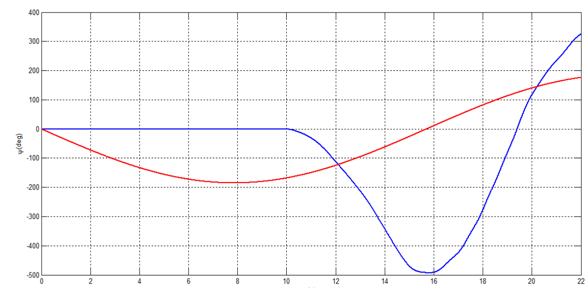

Fig. 12. Yaw trajectories with control based on Euler representation

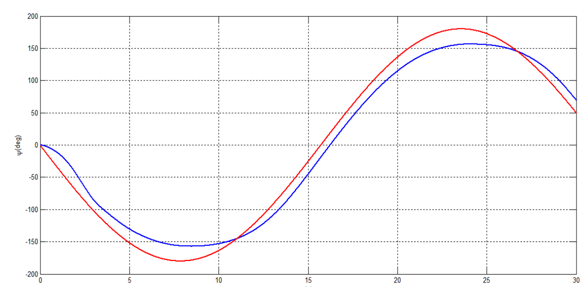

Fig. 13. Yaw trajectories with control based on quaternion representation

VI. Conclusion

In this paper, Super Twisting algorithm based on quaternion has been designed. By this strategy we can avoid singularities, that means, the control can allow the motion of the UAV in all orientation of space, unlike the Euler angles, that cause a control loss when $\phi, \theta = \frac{\pi}{2}$ and $\psi = \pi$. An experimental validation has been done using a Quanser platform. The experimental results prove the stability, roubustness and hight effectivness of the proposed algorithm.

References

[1] Reyes-Valeria, Elias, et al. "LQR control for a quadrotor using unit quaternions: Modeling and simulation." Electronics, Communications and Computing (CONIELECOMP), 2013 International Conference on. IEEE, 2013.

[2] S. Bouabdallah, A. Noth, and R. Siegwart, PID vs LQ control techniques applied to an indoor micro quadrotor, Proc. of the IEEE/RJS International Conference on Intelligent Robots and Systems, vol. 3, pp. 2451-2456, 2004.

[3] Bouabdallah, Samir, and Roland Siegwart. "Backstepping and sliding-mode techniques applied to an indoor micro quadrotor." Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on. IEEE, 2005.

Copyright IPCO-2018
ISSN 2356-5608

[4] Xu, Rong, and Umit Ozguner. ”Sliding mode control of a quadrotor helicopter.” Decision and Control, 2006 45th IEEE Conference on. IEEE, 2006.
[5] L.Derafa, A. Benallegue, and L. Fridman. ”Super twisting control algorithm for the attitude tracking of a four rotors UAV.” Journal of the Franklin Institute 349.2 (2012): 685-699.
[6] Mokhtari M R, Cherki B. A new robust control for minirotorcraft unmanned aerial vehicles. ISA Trans 2015; 56: 86-101.
[7] Lo, Shih-Che, and Yon-Ping Chen. ”Smooth sliding-mode control for spacecraft attitude tracking maneuvers.” Journal of Guidance, Control, and Dynamics 18.6 (1995): 1345-1349.
[8] Tayebi, Abdelhamid, and Stephen McGilvray. ”Attitude stabilization of a VTOL quadrotor aircraft.” IEEE Transactions on control systems technology 14.3 (2006): 562-571.
[9] Wu, Shunan, et al. ”Quaternion-based finite time control for spacecraft attitude tracking.” Acta Astronautica 69.1-2 (2011): 48-58.
[10] Drouot A, Richard E, Boutayeb M. Hierarchical backstepping-based control of a Gun Launched MAV in crosswinds: Theory and experiment Control Engineering Practice 2004; 25 : 16-25
[11] Salas O, Castaneda H, Morales JD. Attitude Observer-Based Robust Control For A Twin Rotor System. Kybernetika, 2013; 49(5): 809-828.
[12] Kendoul F, Yu Z, Nonami K. Guidance and Nonlinear Control System for Autonomous Flight of Minirotorcraft Unmanned Aerial Vehicles. Journal of Field Robotics 2010; 27(3): 311-334.
[13] Lee, Daewon, H. Jin Kim, and Shankar Sastry. ”Feedback linearization vs. adaptive sliding mode control for a quadrotor helicopter.” International Journal of control, Automation and systems 7.3 (2009): 419-428.
[14] Mokhtari M R, Cherki B. Robust Control for Attitude Tracking Problem for a Quadrotor Unmanned Aerial Vehicle. Proceedings of the 3rd International Conference on Systems and Control, Algiers, Algeria, October 29-31, 2013.
[15] Moreno, Jaime A., and Marisol Osorio. ”A Lyapunov approach to second-order sliding mode controllers and observers.” Decision and Control, 2008. CDC 2008. 47th IEEE Conference on. IEEE, 2008.
[16] Choukchou-Braham, Amal, et al. Analysis and control of underactuated mechanical systems. Springer Science Business Media, 2013.
[17] Knoebel, Nathan B., and Timothy W. McLain. ”Adaptive quaternion control of a miniature tailsitter UAV.” American Control Conference, 2008. IEEE, 2008.
[18] Pounds Paul EI, Bersak Daniel R, Dollar Aaron M. Stability of small-scale UAV helicopters and quadrotors with added payload mass under PID control. Auton Robot 2012; 33: 129-142.
[19] DWYER III, THOMAS AW, and Hebertt Sira-Ramirez. ”Variable-structure control of spacecraft attitude maneuvers.” Journal of Guidance, Control, and Dynamics 11.3 (1988): 262-270.
[20] Dwyer, Thomas AW, and Jinho Kim. ”Bandwidth-limited robust nonlinear sliding control of pointing and tracking maneuvers.” American Control Conference, 1989. IEEE, 1989.

Copyright IPCO-2018
ISSN 2356-5608